

## Problem 2.26

[Difficulty: 4]

**2.26** Consider the garden hose of Fig. 2.5. Suppose the velocity field is given by  $\vec{V} = u_0\hat{i} + v_0\sin[\omega(t - x/u_0)]\hat{j}$ , where the  $x$  direction is horizontal and the origin is at the mean position of the hose,  $u_0 = 10$  m/s,  $v_0 = 2$  m/s, and  $\omega = 5$  cycle/s. Find and plot on one graph the instantaneous streamlines that pass through the origin at  $t = 0$  s, 0.05 s, 0.1 s, and 0.15 s. Also find and plot on one graph the pathlines of particles that left the origin at the same four times.

**Given:** Velocity field

**Find:** Plot streamlines that are at origin at various times and pathlines that left origin at these times

**Solution:**

For streamlines

$$\frac{v}{u} = \frac{dy}{dx} = \frac{v_0 \cdot \sin\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right]}{u_0}$$

So, separating variables ( $t = \text{const}$ )

$$dy = \frac{v_0 \cdot \sin\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right]}{u_0} \cdot dx$$

Integrating

$$y = \frac{v_0 \cdot \cos\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right]}{\omega} + c$$

Using condition  $y = 0$  when  $x = 0$

$$y = \frac{v_0 \cdot \left[ \cos\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right] - \cos(\omega \cdot t) \right]}{\omega}$$

This gives streamlines  $y(x)$  at each time  $t$

For particle paths, first find  $x(t)$

$$\frac{dx}{dt} = u = u_0$$

Separating variables and integrating

$$dx = u_0 \cdot dt \quad \int_r^o \quad x = u_0 \cdot t + c_1$$

Using initial condition  $x = 0$  at  $t = \tau$

$$c_1 = -u_0 \cdot \tau \quad x = u_0 \cdot (t - \tau)$$

For  $y(t)$  we have

$$\frac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right] \quad \text{so} \quad \frac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left[t - \frac{u_0 \cdot (t - \tau)}{u_0}\right]\right]$$

and

$$\frac{dy}{dt} = v = v_0 \cdot \sin(\omega \cdot \tau)$$

Separating variables and integrating

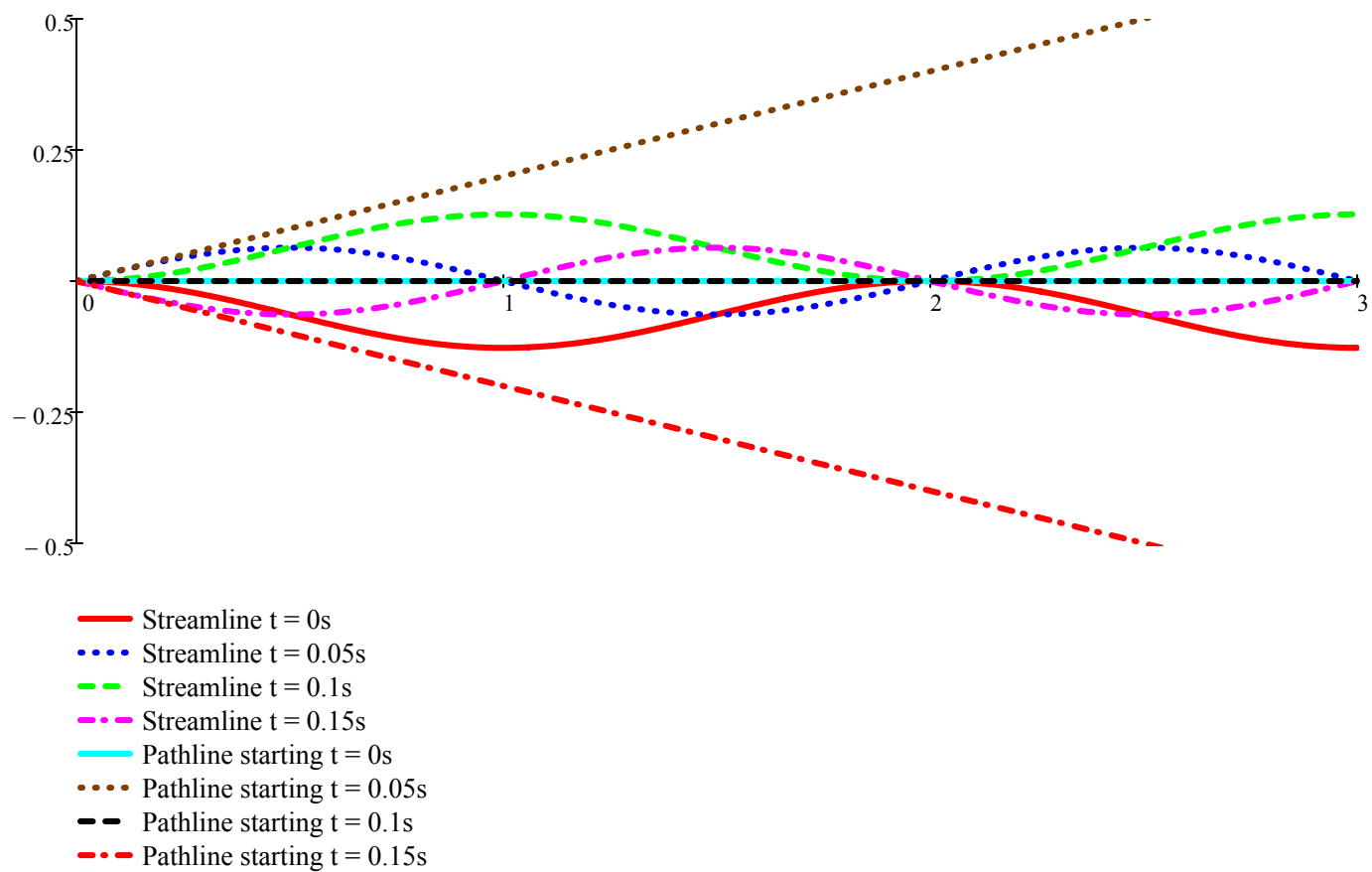
$$dy = v_0 \cdot \sin(\omega \cdot \tau) \cdot dt \quad y = v_0 \cdot \sin(\omega \cdot \tau) \cdot t + c_2$$

Using initial condition  $y = 0$  at  $t = \tau$

$$c_2 = -v_0 \cdot \sin(\omega \cdot \tau) \cdot \tau \quad y = v_0 \cdot \sin(\omega \cdot \tau) \cdot (t - \tau)$$

The pathline is then

$$x(t, \tau) = u_0 \cdot (t - \tau) \quad y(t, \tau) = v_0 \cdot \sin(\omega \cdot \tau) \cdot (t - \tau) \quad \text{These terms give the path of a particle } (x(t), y(t)) \text{ that started at } t = \tau.$$



The streamlines are sinusoids; the pathlines are straight (once a water particle is fired it travels in a straight line).

These curves can be plotted in *Excel*.